ST. MARY'S DOMINICAN HIGH SCHOOL
Calculus BC (AP) Summer Worksheet

Dear Student:_____(name)

Your progress in Calculus BC is dependent upon knowledge of certain Algebra and Trigonometry concepts. In order to help you prepare for this course, solve the problems on this worksheet, and check your answers with the attached answer key.

In addition, complete the problems listed below from Appendices D-1 and D-3.
The solutions are included in the folder.

Complete: D.1 # 25-34, 36, 41, 43, 44, 49, 51, 53
              D.3 # 11a, 14b, 18, 27a, 31-35, 37, 38, 40, 51, 61

If you have difficulty with any of the concepts, please review them during the summer.

In order to receive full credit for your work, be sure to follow these instructions.

1.   Do all work IN PENCIL and on loose-leaf paper.
2.   Copy each problem. **SHOW ALL REQUIRED WORK NEATLY AND IN A CLEAR MANNER.** Box your answer. Word problems should be answered in complete sentences.
3.   Bring your completed assignment to school on the second day of class.

Your work will be collected, and a grade will be given based on **completeness and effort**. Try your best!!!
If you need assistance, consult the video tutors available khanacademy.org,

Sincerely,

St. Mary's Dominican Mathematics Department

Answer Key

Part I

1.	E	9.	B
2.	A	10.	C
3.	B	11.	C
4.	C	12.	E
5.	A	13.	A
6.	D	14.	B
7.	C	15.	D
8.	D		

Part II

1. a. $2\sqrt{x}(1+3x)$
   b. $e^{-x}(1-x+2x^2)$
   c. $\sin x(1+\sec x)$
   d. $\dfrac{1}{2x}\left(\dfrac{1}{x+2}\right)$

2. a. $(x-1)^3$
   b. $(x^2+5)(x+\sqrt{5})(x-\sqrt{5})$
   c. $(x-2)^4$

Part II, continued

3. a. $(\cos x + \sin x)(5\cos x - 5\sin x + 1)$
   b. $(\cos x + 2 + \tan x)(\cos x + 2 - \tan x)$

4. a. $(x+1)(2x^2 - 2x + 7)$
   b. $(x-2)(x^3 - x^2 - x - 1)$

5. a. $(x-1)(x^2 - x - 1)$
   b. $3(x-2)(x+2)(\sqrt{7}+x)(\sqrt{7}-x)$
   c. $(x^2 - 1 - x^2) = -1$
   d. $(x-3)^2(x-2)^2(x-5)$
   e. $(1-x)$

6. a. $\dfrac{x^2 - 3x - 1}{(x-1)^2}$
   b. $x^{\frac{1}{6}}\left(x^{\frac{1}{6}} - 1\right)$
   c. $(2-x)$
   d. $\cos x$

7. a. $2 \pm \sqrt{5}$ b. 1;-1.5   c. π

8. a. $(x-2)(x-3)(2x-3)$
   b. $(3x+1)(3x-2)(x+1)$

9. a. 4
   b. $\dfrac{x^2}{(x^2+1)^{\frac{3}{2}}}$
   c. $\dfrac{-4}{(x+1)(x-1)}$

10. a. $\dfrac{1+\cos x}{\sin^2 x}$
    b. $-\dfrac{1+\sqrt{x^2+1}}{x}$
    c. $2\sqrt{x^2+1} - 2x$

Part III:

1. DNE

2. $\dfrac{1}{2}$

3. 0

4. $\dfrac{32}{5}$

5. a. convergent by ratio test
   b. divergent by ratio test
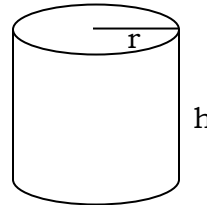   c. convergent by comparison test

Part III, continued:

6.   a. $\dfrac{\pi}{2}$ b. $\dfrac{\pi}{2}$ c. $\dfrac{1}{2}$ d. $-\dfrac{1}{2}$

7.   a. $\dfrac{1}{x-6}+\dfrac{1}{x+1}$

       b. $\dfrac{6}{x}-\dfrac{2}{x^2}-\dfrac{6}{x+1}+\dfrac{3}{(x+1)^2}-\dfrac{9}{(x+1)^3}$

       c. $\dfrac{4x}{x^2+6}+\dfrac{2x-4}{(x^2+6)^2}$

PART I:

1. The x-coordinate of the point of intersection of $-2x+y=6$ and $x+y=-3$ is

     a. 3   b. 0   c. -1   d. -2   e. -3

2. The right circular cylinder shown on the right has a circular base and an open top. Its surface area in terms of r and h is

     a. $\pi r^2 + 2\pi rh$
     b. $4\pi r + 2h$
     c. $\pi r^2 h$
     d. $2\pi r + h$
     e. $2\pi r^2 + 2\pi rh$

3. If $\log_5(x+8)=2$ then $x=$

     a. 18   b. 17   c. 10   d. 2   e. $\dfrac{2}{\log_5 2}-8$

4. $(32)^{1/5}(9)^{-1/2}=$

     a. 6   b. $\dfrac{3}{2}$ c. $\dfrac{2}{3}$ d. $(288)^{-1/10}$ e. -6

5. If $\dfrac{(3x+1)(x-3)}{x+4}=0$ then $x=$

     a. 3 or $-\dfrac{1}{3}$ b. 3, -4, or $-\dfrac{1}{3}$ c. 3 or -4   d. -3 or $\dfrac{1}{3}$ e. -3, 4, or $\dfrac{1}{3}$

6. In a standard coordinate system, the graph of the equation $y=-6x+7$ is
     a. A vertical line
     b. A horizontal line
     c. Not a line
     d. A line falling to the right
     e. A line rising to the right

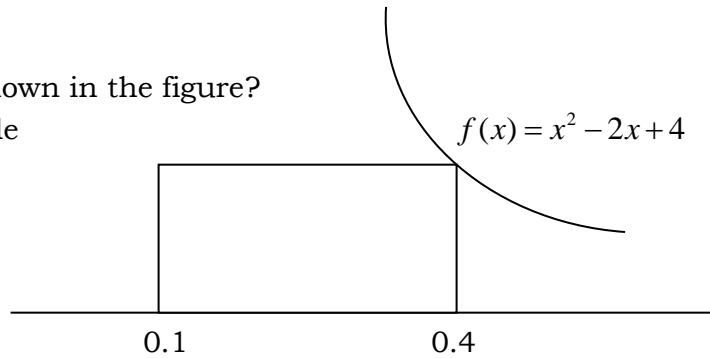7. The quantity $p+q$ is a factor of how many of the following?

$$p^2 - q^2 \qquad p^2 + q^2 \qquad p^3 - q^3 \qquad p^3 + q^3$$

    a. None   b. one only   c. two only   d. three only   e. four

8. What is the area of the rectangle shown in the figure?
   Note: the figure is not drawn to scale

   $f(x) = x^2 - 2x + 4$

       a. 0.04
       b. 0.1008
       c. 0.3
       d. 1.008
       e. 3.36

   0.1   0.4

9. If $f(x) = \dfrac{4x+2}{x+4}$ then $f(b+1) =$

       a. $\dfrac{6}{5}$ b. $\dfrac{4b+6}{b+5}$ c. $\dfrac{4b+3}{b+5}$ d. $\dfrac{4b+2}{b+5}$ e. $\dfrac{4b+2}{b+4}$
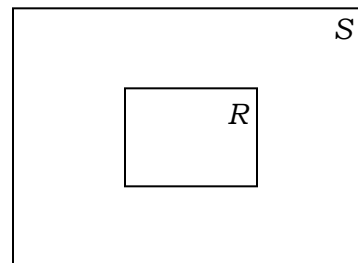
10. The length of a certain rectangle is 3 meters more than twice its width. What is the width of the rectangle if the perimeter of the rectangle is 186 meters?

       a. 9m   b. 18m   c. 30m   d. 32m   e. 61m

11. A rectangle R has width x and length y. A rectangle S is formed from R by multiplying each of the sides of the rectangle R by 8 as shown in the figure at the right. What is the area of the portion of S lying outside of R?
    Note: the figure is not drawn to scale

        a. xy
        b. $8xy$
        c. $63xy$
        d. $64xy$
        e. $x^8 y^8$

    S R

12. $\sec\theta \cot\theta \sin^2\theta =$

        a. $\cos\theta$ b. $\csc\theta$ c. $\cot\theta$ d. $\sec\theta$ e. $\sin\theta$
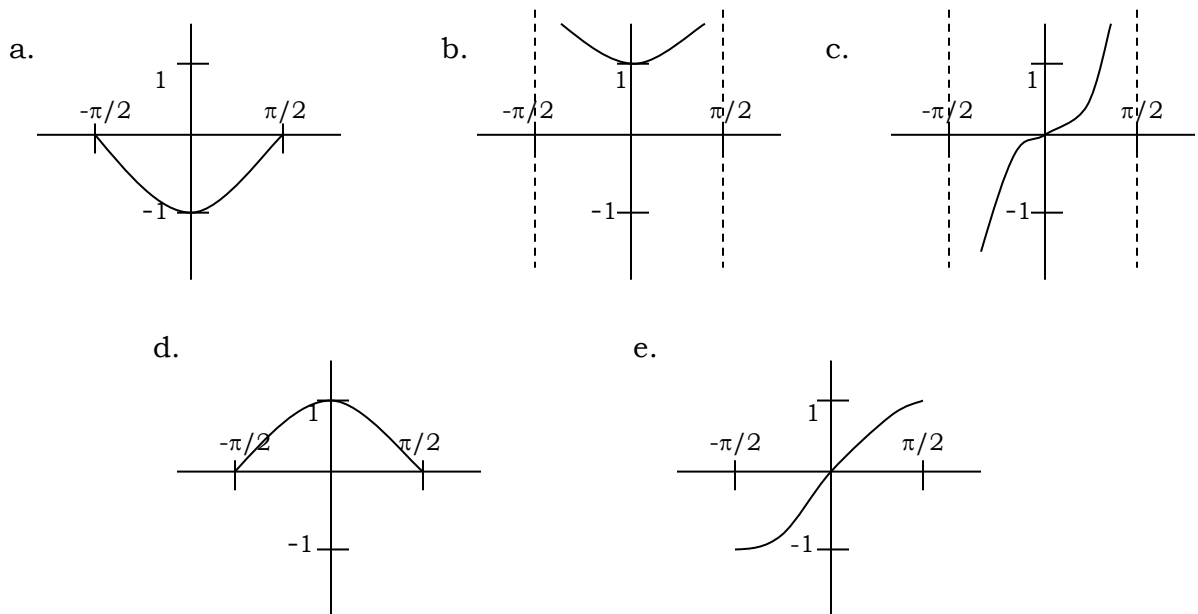
13. If $f(x) = \cos(3x)$, then $f\left(\dfrac{\pi}{6}\right) =$

    a. 0   b. $\dfrac{1}{2}$ c. $\dfrac{1}{\sqrt{2}}$ d. $\dfrac{\sqrt{3}}{2}$ e. 1

14. $\sin^2 \theta - 1 =$

    a. $-\cos\theta$ b. $-\cos^2\theta$ c. $-\csc^2\theta$ d. $\cos^2\theta$ e. $-\cos 2\theta$

15. Of the following, which best represents the graph of $y = \cos x$ for x between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$?

a.   b.   c.

d.   e.

PART II:

1. Factor out monomial factors as indicated.

    a. $2\sqrt{x} + 6x^{3/2} = 2\sqrt{x}\left(\qquad\right)$

    b. $e^{-x} - xe^{-x} + 2x^2 e^{-x} = e^{-x}\left(\qquad\right)$

    c. $\sin x + \tan x = \sin x\left(\qquad\right)$

    d. $\dfrac{1}{2x^2 + 4x} = \dfrac{1}{2x}\left(\qquad\right)$

2. Factor these special products into linear or irreducible quadratic factors.

    a. $x^3 - 3x^2 + 3x - 1$

    b. $x^4 - 25$

    c. $x^4 - 8x^3 + 24x^2 - 32x + 16$

3. Factor completely by grouping.
   a. $5\cos^2 x - 5\sin^2 x + \sin x + \cos x$

   b. $\cos^2 x + 4\cos x + 4 - \tan^2 x$

4. Use synthetic division to factor as indicated.
   a. $2x^3 + 5x + 7 = (x+1)($ $\quad\quad)$

   b. $x^4 - 3x^3 + x^2 + x + 2 = (x-2)($ $\quad\quad)$

5. Factor completely.

   a. $(x-1)^2 (x) - (x-1) =$

   b. $3(x^2 - 4)(x^2 - 1) - 6(x^2 - 4)^2 =$

   c. $\sqrt{x^2 - 1} - \dfrac{x^2}{\sqrt{x^2 - 1}} = \dfrac{1}{\sqrt{x^2 - 1}}($ $\quad\quad)$

   d. $(x-3)^3 (x-2)^2 - 2(x-3)^2 (x-2)^2 =$

   e. $(2x-1)^{3/2} (x^{1/2}) - (2x-1)^{5/2} (x^{-1/2}) = (2x-1)^{3/2} (x)^{-1/2} ($ $\quad\quad)$

6. Reduce each expression to lowest terms.

   a. $\dfrac{(x-1)^3 (x-2) - 3(x-1)^2}{(x-1)^4}$

   b. $\dfrac{x^{\frac{1}{2}} - x^{\frac{1}{3}}}{x^{\frac{1}{6}}}$

   c. $\dfrac{\sqrt{x-1} - (x-1)^{\frac{3}{2}}}{\sqrt{x-1}}$

   d. $\dfrac{1 - (\sin x - \cos x)^2}{2\sin x}$

7. Solve using the quadratic formula.

    a. $x^2 - 4x - 1 = 0$

    b. $2x^2 + x - 3 = 0$

    c. $\cos^2 x + 3\cos x + 2 = 0$

8. Factor completely—some may require synthetic division.

    a. $2x^3 - 13x^2 + 27x - 18$

    b. $9x^4 - 3x^3 + 7x^2 - 3x - 2$

9. Simplify these expressions.

    a. $\dfrac{(x-1)(x+3)-(x-1)^2}{x-1}$

    b. $\dfrac{\sqrt{x^2+1}-\dfrac{1}{\sqrt{x^2+1}}}{x^2+1}$

    c. $\dfrac{1}{x+1}-\dfrac{1}{x-1}-\dfrac{2}{x^2-1}$

10. Rationalize.

    a. $\dfrac{1}{1-\cos x}$

    b. $\dfrac{x}{1-\sqrt{x^2+1}}$

    c. $\dfrac{2}{x+\sqrt{x^2+1}}$

1. $\lim\limits_{n\to\infty} \dfrac{2+3n^2-5n^3}{n^2-n+6}$

2. $\lim\limits_{n\to\infty} \dfrac{2n^2-n-1}{4n^2+3}$

3. $\lim\limits_{n\to\infty} \dfrac{(n+1)(2n-3)}{n^3-2}$

4. Find the sum of the series, if possible. If not explain why: $8-2+\dfrac{1}{2}-\dfrac{1}{8}+...$

5. Determine if the following series are convergent or divergent, explain why:

    a. $3+\dfrac{3^2}{2!}+\dfrac{3^3}{3!}+\dfrac{3^4}{4!}+...$

    b. $\dfrac{2}{3}+\dfrac{2^2}{5}+\dfrac{2^3}{7}+...$

    c. $1+\dfrac{1}{3^2}+\dfrac{1}{5^2}+\dfrac{1}{7^2}+...$

6. Solve the following with Principal Values only:

    a. $\arccos 0$

    b. $\sin^{-1}\left(\tan\dfrac{\pi}{4}\right)$

    c. $\cos\left(\tan^{-1}\sqrt{3}\right)$

    d. $\cos\left(\arccos 0+\arcsin\dfrac{1}{2}\right)$

7. Solving using partial decomposition of fractions:

    a. $\dfrac{2x-5}{x^2-5x-6}$

    b. $\dfrac{7x^3-2}{x^2(x+1)^3}$

    c. $\dfrac{4x^3+26x-4}{\left(x^2+6\right)^2}$