### ST. MARY'S DOMINICAN HIGH SCHOOL Calculus and Advanced Math II Summer Worksheet

Dear student:\_

\_\_(name)

The purpose of this summer worksheet is to help you practice your math skills while providing examples of ACT/PSAT-type questions. The questions were specifically chosen because they relate to topics that will be used in your math class next year or because they cover material you have learned that is on the ACT/PSAT.

# In order to receive full credit for your work, be sure to follow these instructions.

- 1. Print out the worksheet (print on both sides) and show all work neatly and in a concise manner next to each question- in PENCIL! If work can not be shown, explain how you were able to solve the problem or what strategy you used (process of elimination, guess and check, graphing calculator, etc.). <u>All problems must have work or an explanation!</u>
- 2. Check your answers with the answer key provided. Review and re-try any questions you missed. If you have difficulty with any of the concepts, please review them during the summer.
- 3. Bring your completed assignment to school on the second day of class.

## **Standardized Test-Taking Tips:**

- 1. Underline or circle key numbers/words in the problem.
- 2. Read the question that is being asked. Many times the answer to the question is not necessarily the solution to the equation.
- 3. As you eliminate choices, scratch them out.
- 4. Use these strategies as they apply:
  - Guess and check with the choices (start with the middle choice)
  - Assign numbers to represent variables in the question
  - Start at the end and work backwards
  - Look for a pattern on a simpler problem
  - Use a graphing calculator to assist when needed

# Note: On the ACT, unless otherwise stated, all of the following should be assumed:

- 1. Illustrative figures lie in a plane.
- 2. Geometric figures lie in a plane.
- 3. The word "line" indicates a straight line.
- 4. The word "average" indicates the arithmetic mean.
- Choices are usually listed from either greatest to least or least to greatest.

You may use your calculator for any problems you choose, but some of the problems may best be done without using a calculator. Reference the "ACT Math Formulas" file in School Links and Files.

Your work will be collected, and a grade will be given based on <u>completeness and effort</u>. Try your best!!! If you need assistance, consult the video tutors available at khanacademy.org or <u>https://academy.act.org/summer-slide-educator/</u>. For more individualized ACT practice, log in to act.org/academy and input your ACT subscores.

## Sincerely,

St. Mary's Dominican Mathematics Department

## Don't forget – you must show work next to each problem!!! If you use "Guess and Check," show your guesswork.

# Pre-Algebra

- 1. A typical high school student consumes 67.5 pounds of sugar per year. As part of a new nutrition plan, each member of a track team plans to lower the sugar he or she consumes by at least 20% for the coming year. Assuming each track member had consumed sugar at the level of a typical high school student and will adhere to this plan for the coming year, what is the maximum number of pounds of sugar to be consumed by each track team member in the coming year?
  - A. 14
  - **B.** 44
  - C. 48
  - D. 54
  - E. 66
- 2. The five integers in a set are 121, 118, 133, 142, and *x*. Which of the following could be equal to *x* if *x* is the median number in the set?
  - A. 120
  - B. 129
  - C. 133
  - D. 135
  - E. 144
- 3. Joelle earns her regular pay of \$7.50 per hour for up to 40 hours of work in a week. For each hour over 40 hours of work in a week, Joelle is paid 1.5 times her regular pay. How much does Joelle earn for a week in which she works 42 hours?
  - A. \$126.00
  - B. \$315.00
  - C. \$322.50
  - D. \$378.00
  - E. \$472.50
- 4. Jerome, Kevin, and Seth shared a submarine sandwich. Jerome at  $\frac{1}{2}$  of the sandwich,

Kevin ate  $\frac{1}{3}$  of the sandwich, and Seth ate the rest. What is the ratio of Jerome's share to Kevin's share to Seth's share?

- A. 3:6
- B. 2:6:3
- C. 2:3:1
- D. 3:2:1
- E. 6:3:2

- 5. How many possible combinations of answers can one have for answers on a 5-question True/False test?
  - A. 2
  - B. 4
  - C. 8
  - D. 16
  - E. 32
- 6. The table below gives the number of questions answered correctly on the 20-question written section of a driver education test.

Number Correct	12	13	14	15	16	17	18	19	20
Frequency	2	3	0	3	3	5	5	6	4

What is the mode of the data?

- A. 3
- B. 6
- C. 18
- D. 19
- E. 20
- 7. While away at school, Eileen receives an allowance of \$400 each month, 35% of which she uses to pay her bills. If she budgets 30% of the remainder for shopping, allots \$130 for entertainment, and saves the rest of the money, what percentage for her allowance is she able to save each month?
  - A. 2.5%
  - B. 13%
  - C. 20%
  - D. 35%
  - E. 52%
- 8. 40% of 250 is equal to 60% of what number?
  - A. 150
  - **B.** 160
  - C.  $166\frac{2}{3}$
  - D. 270
  - E. 375

#### **Elementary Algebra**

- 9. Only 10<sup>th</sup>, 11<sup>th</sup>, and 12<sup>th</sup> grade students attend Lincoln High School. The ratio of 10<sup>th</sup> graders to the school's total student population is 86:255, and the ratio of 11<sup>th</sup> graders to the school's total student population is 18:51. If 1 student is chosen at random from the entire school, which grade is that student most likely to be in?
  - A. 10<sup>th</sup>
  - B.  $11^{\text{th}}$
  - $C. \ 12^{th}$
  - D. All grades equally likely
  - E. Cannot be determined from the given information
- 10. A dog eats 7 cans of food in 3 days. At this rate, how many cans of food does the dog eat in 3 + d days?
  - A.  $\frac{7}{3} + d$ B.  $\frac{7}{3} + \frac{d}{3}$ C.  $\frac{7}{3} + \frac{7}{3d}$ D.  $7 + \frac{d}{3}$ E.  $7 + \frac{7d}{3}$
- 11. Which of the following ordered pairs (*x*, *y*) satisfies both equations  $y = x^2 + 3x 4$  and x = y 4?
  - A. (0, -4)
  - B. (2, 6)
  - C. (3, 14)
  - D. (5, 9)
- 12. A line is graphed in the *xy*-plane. If the line has a negative slope and a positive *y*-intercept, which of the following points cannot lie on the line?
  - A. (-3, -3)
    B. (-3, 3)
    C. (3, -3)
    D. (3, 3)

- 13. There are 18 boys and 10 girls in Mr. Jackson's class. If Mr. Jackson chooses a student at random, what is the probability that the student is a girl?
  - A.  $\frac{5}{14}$  $\frac{5}{9}$ B.  $\frac{9}{14}$ С.  $\frac{4}{5}$   $\frac{9}{5}$ D. E.
- 14. The table below shows the monthly electricity bills of Joseph and Samuel for the first five months of a year.

Electricity Bills						
Month	Joseph	Samuel				
January	\$184.66	\$188.99				
February	\$193.12	\$181.27				
March	\$175.99	\$176.35				
April	\$145.30	\$149.23				
May	\$180.33	\$185.66				

.... - 4 - 12 - 12 4 -- D:11

Based on the information in the table, which of these statements is true about the ranges and medians of the bills?

- A. Both the range and median of Joseph's bills are less than the range and median of Samuel's bills.
- B. Both the range and median of Joseph's bills are greater than the range and median of Samuel's bills.
- C. The range of Joseph's bills is less than the range of Samuel's bills, while the median of Joseph's bills is greater than the median of Samuel's bills.
- D. The range of Joseph's bills is greater than the range of Samuel's bills, while the median of Joseph's bills is less than the median of Samuel's bills.
- 15. A bag contains 12 red marbles, 5 yellow marbles, and 15 green marbles. How many additional red marbles must be added to the 32 marbles already in the bag so that the

probability of randomly drawing a red marble is  $\frac{3}{5}$ ? Hint: Strategy: Guess and Check

- A. 13
- B. 18
- C. 28
- D. 32
- E. 40

16. If 2x + y = 6 and x + 3y = -2, then x - 2y = ?

- A. -12
- B. 0
- C. 4
- D. 8
- E. 12
- 17. Which of the following statements completely describes the solution set for the inequality 2(x-3) > 3(x+4)?
  - A. *x* < −18
  - B. *x* < 18
  - C. *x* > 18
  - D. x > -18
  - E. There are no solutions for x.
- 18. If *t* is 5 more than *x*, and *x* is 3 less than *r*, what is *t* when r = 3?
  - A. -5
  - B. -2
  - C. 1
  - D. 5
  - E. 8

19. If the average of 292, 305, 415, and *x* is 343, what is the value of *x*?

- A. 315
- B. 339
- C. 360
- D. 364
- E. 382
- 20. In a math class, the midterm is worth 30%, the final exam is worth 50%, and a class project is worth 20%. If Jason scored 86% on the midterm, 95% on the final, and 89% on the project, what was his final grade in the class, rounded to the nearest integer?
  - A. 90
  - B. 91
  - C. 92
  - D. 93
  - E. 94

- 21. The expression  $3x^2y(xy^2 + 4x^3y)$  is equivalent to which of the following?
  - A. 3xy + 12xB.  $xy^2 + 4x^3y$ C.  $15x^8y^5$ D.  $3x^3y^3 + 12x^5y^2$
  - E.  $3xy^2 + 12x^3y$
- 22. When 2x is subtracted from 48 and the difference is divided by x + 3, the result is 4. What is the value of x?
  - A. 2
  - B. 5
  - C. 6
  - D. 8
  - E. 12

23. If  $3^5 = x$ , which of the following expressions is equal to  $3^{11}$ ?

- A. 243*x*
- B.  $3x^2$
- C.  $9x^4$
- D.  $27x^3$
- E.  $x^6$

## **Plane Geometry**

- 24. The angle of elevation from a point on the ground to the top of a building is 37°. The point is 75 feet away from the building. Which of the following is closest to the height, in feet, of the building?
  - A. 45
  - B. 57
  - C. 60
  - D. 94
  - E. 125

25. What is the volume, in cubic centimeters, of a cube if the area of 1 square face is  $144cm^2$ ?

A. 36
B. 1,728
C. 20,736
D. 46,656
E. 373,248

26. In the figure shown below, the measure of  $\angle BAC$  is  $(x+20)^{\circ}$  and the measure of  $\angle BAD$  is



27. The length of the hypotenuse of right triangle RST is 16. If the measure of  $\angle R = 30^{\circ}$ , what is the length of RS?







- 28. The radius of a right circular cone is doubled and its height is tripled to form a new right circular cone. What is the ratio of the volume of the original cone to the volume of the new cone?  $\left(V=1/3\pi r^2h\right)$ Hint: Strategy: Assign values to represent the radius and the height.
  - A. 1:6
  - B. 1:12
  - C. 1:18
  - D. 1:36
  - E. 1:216

29. The square below is divided into 3 rows of equal area. In the top row, the region labeled A has the same area as the region labeled B. In the middle row, the 3 regions have equal areas. In the bottom row, the four regions have equal areas. What fraction of the square's area is in a region labeled A? Hint: Strategy: Assign values to represent each region.

A.	$\frac{1}{9}$	1	A	]	3
В.	$\frac{3}{9}$ $\frac{6}{9}$	А		В	С
C.	$\frac{6}{9}$	А	В	C	D
D.	$\frac{13}{12}$			I	1
E.	$\frac{13}{36}$				

30. Circle O has a radius of 5 and  $m \angle AOB = 45^{\circ}$ . What is the length of arc AB?



- 31. A line intersects two parallel lines, forming 8 angles. If one of the angles has measure  $a^{\circ}$ , how many of the other 7 angles are supplementary to it?
  - A. 0
  - B. 1
  - C. 2 D. 3
  - D. 3 E. 4
- 32. What is the perimeter, in inches, of the isosceles right triangle whose hypotenuse is  $8\sqrt{2}$  inches long?
  - A. 8 B.  $8+8\sqrt{2}$ C.  $8+16\sqrt{2}$ D. 16 E.  $16+8\sqrt{2}$

## **Intermediate Algebra**

33. Students studying motion observed a cart rolling at a constant rate along a straight line. The table below gives the distance, d feet, the cart was from a reference point at 1-second intervals from t = 0 seconds to t = 5 seconds.

t	0	1	2	3	4	5
d	14	20	26	32	38	44

Which of the following equations represents this relationship between *d* and *t*?

A. d = t + 14B. d = 6t + 8C. d = 6t + 14D. d = 14t + 6E. d = 34t

- 34. As part of a probability experiment, Ellie is to answer 4 multiple-choice questions. For each question, there are 3 possible answers, only 1 of which is correct. If Ellie randomly and independently answers each question, what is the probability that she will answer the 4 questions correctly?
  - A.  $\frac{27}{81}$

  - B.  $\frac{12}{81}$

  - C.  $\frac{4}{81}$
  - D.  $\frac{3}{81}$
  - E.  $\frac{1}{81}$
- 35. In the equation below, *a* is a constant. For what value of *a* does the equation have an infinite number of solutions?

$$32x - 20 = 4(8x + a)$$

A. -5

- B. 2
- C. 5
- D. 10

- 36. If a > b, then |a b| + |b a| is equal to what expression? Hint: Strategy: Assign values to represent the variables.
  - A. 0
    B. 2a
    C. 2b
    D. 2a + 2b
  - E. 2a 2b

37. If function f is defined by  $f(x) = 3x^2 - 5x + 4$ , what is f(x-4)?

A.  $f(x-4) = 3x^2 - 5x$ B.  $f(x-4) = 3x^2 - 5x + 72$ C.  $f(x-4) = 3x^2 - 29x + 52$ D.  $f(x-4) = 3x^2 - 29x + 72$ 

38. The determinant of a matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$  equals ad - bc. What must the value of x for the matrix  $\begin{bmatrix} x & 8 \\ x & x \end{bmatrix}$  to have a determinant of -16? A. -4B. -2C.  $-\frac{8}{5}$ D.  $\frac{8}{3}$ E. 4

39. Find the points at which the graphs of  $y = \frac{1}{2}x^2 - 3$  and y = x + 1 intersect.

A. 
$$(-2, -3)$$
 and  $(4, 5)$   
B.  $(-1, -2)$  and  $(5, 4)$   
C.  $(-4, -3)$  and  $(-2, 4)$   
D.  $(-4, -3)$  and  $(2, 3)$   
E.  $(-2, -1)$  and  $(4, 5)$ 

- 40. Which value is *not* in the domain of  $f(x) = \sqrt{x-5}$ ?
  - A. 4
  - B. 5
  - C. 6
  - D. 7
  - E. 8.5

41. What is the range of  $f(x) = -(x-2)^2 - 4?$ 

- A. All real numbers
- B.  $y \ge -8$
- C.  $y \le -8$
- D.  $y \ge -4$
- E.  $y \leq -4$
- 42. In the triangle below, each row has two more asterisks than the previous row.

\* \*\*\* \*\*\*\*\* \*\*\*\*\*

If the triangle continues with the same pattern, how many asterisks will be in the 50<sup>th</sup> row?

A. 151
B. 121
C. 99
D. 98
E. 97

43. If  $\log_b 3 = x$  and  $\log_b 5 = y$ , then  $\log_b 45 = ?$ 

- A. 2x + yB. x + 2yC. 2xy
- D.  $2x^2y$
- E.  $x^2y$

44.  $2i^4 - i^6 = ?$  Hint: Recall  $i^2 = -1$ .

A. *i*−1

- B. *i*+1
- C. -3
- D. -2
- E. 3

45. The table of values shown is for some linear function f(x). Find f(10).

	x	у
	-2	-11
	-1	-7
	0	-3
	1	1
А.	7	
В.	10	
С.	21	
D.	37	
Е.	43	

- 46. A geometric sequence has a first term of x y and a common ratio of x + y. What is the third term of the sequence?
  - A.  $x^{3} + 2x^{2}y y^{3}$ B.  $x^{3} + x^{2}y - xy^{2} - y^{3}$ C.  $x^{3} - y^{3}$ D.  $x^{2} - y^{2}$ E.  $x^{2} + y^{2}$

47. Which of the following gives the solution to the inequality |x-3|+6<15?

A. -6 < x < 12B. -6 < x < 9C. -9 < x < 9D. x < -9E. x < -6 48. If  $\frac{p+q}{s} = 9$ ,  $\frac{q}{p} = 4$ , and  $\sqrt{q} = 6$ , what is the value of *s*? A.  $\frac{5}{6}$ B. 5 C. 9 D. 13 E. 36

#### **Coordinate Geometry**

- 49. A circle is graphed in the *xy*-plane. If the circle has a radius of 3 and the center of the circle is at (4, -2), which of the following could be an equation of the circle?
  - A.  $(x+4)^{2} + (y-2)^{2} = 3$ B.  $(x+4)^{2} - (y-2)^{2} = 3$ C.  $(x-4)^{2} + (y+2)^{2} = 9$ D.  $(x-4)^{2} - (y+2)^{2} = 9$
- 50. Liquid going through a cooling system is chilled so that its temperature decreases at a constant rate from 100° to 25° in 5 seconds. Which of the following functions represents the temperature C, in degrees Celsius, as a function of the time t, in seconds, after chilling began?
  - A. C = -25 + 15tB. C = 25 - 15tC. C = 25 + 15tD. C = 100 - 15t
- 51. What is the distance in the standard (*x*, *y*) coordinate plane between the points (-3, 6) and (-2, -1)?
  - A.  $\sqrt{26}$
  - B.  $\sqrt{74}$
  - C.  $2\sqrt{5}$
  - D.  $5\sqrt{10}$
  - E.  $5\sqrt{2}$

52. Consider the following lines in the standard (x, y) coordinate plane:

3x + y = 10y - kx = 4

What value of *k* would make the lines perpendicular to each other?

- A. -3B.  $-\frac{1}{3}$ C.  $\frac{1}{3}$ D.  $\frac{2}{3}$ E. 3
- 53. Line *l* has an undefined slope and contains the point (-2, 3). Which of the following points is also on line *l*?
  - A. (0, 3)
  - B. (5, 5)
  - C. (0, 0)
  - D. (3, -2)
  - E. (-2, 5)

# **Trigonometry**

54. In which quadrant must  $\theta$  lie if  $\cos \theta > 0$  and  $\sin \theta < 0$ ?

- A. I
- B. II
- C. III
- D. IV
- E. No such angle exists

55. The sides of an acute triangle measure 14cm, 18cm, and 20cm, respectively. Which of the following equations, when solved for  $\theta$ , gives the measure of the smallest angle of the triangle?

Recall: Law of Sines:  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$  Law of Cosines:  $c^2 = a^2 + b^2 - 2ab \cos C$ A.  $\frac{\sin \theta}{14} = \frac{1}{18}$ B.  $\frac{\sin \theta}{14} = \frac{1}{20}$ C.  $\frac{\sin \theta}{20} = \frac{1}{14}$ D.  $14^2 = 18^2 + 20^2 - 2(18)(20) \cos \theta$ E.  $20^2 = 14^2 + 18^2 - 2(14)(18) \cos \theta$ 

56. What is the measure of the angle  $\frac{3\pi}{4}$  expressed in degrees?

- A. 45°
- B. 60°
- C. 120°
- D. 135°
- E.  $270^{\circ}$
- 57. According to the measurements given in the figure below, which of the following expressions gives the distance, in miles, from the boat to the dock?



58. The lengths in feet of the sides of a right triangle PQR are as shown in the diagram below, with  $x \ge 0$ . What is the cosecant of *P* in terms of *x*?



59. If  $\cos\theta = \frac{12}{13}$ , and  $\theta$  is located in Quadrant IV, then  $\tan\theta = ?$ 



60. What is the period of  $y = 3\sin 2x$ ?

A.  $\frac{\pi}{2}$ 

- B. 2C. 3
- D.  $\pi$
- E. 2π

# Answer Key

1. D 2. B 3. C 4. D 5. E 6. D 7. B 8. C 9. B	<ul> <li>21. D</li> <li>22. C</li> <li>23. B</li> <li>24. B</li> <li>25. B</li> <li>26. B</li> <li>27. C</li> <li>28. B</li> <li>29. E</li> </ul>	41. E 42. C 43. A 44. E 45. D 46. B 47. A 48. B 49. C
10. E	<b>30.</b> D	50. D
11. B	31. E	51. E
12. A	32. E	<b>52.</b> C
13. A	<b>33.</b> C	53. E
14. <b>D</b>	34. E	54. D
15. B	35. A	55. D
16. D	36. E	56. D
17. A	37. D	57. B
18. D	38. E	58. A
19. C	39. E	59. B
20. B	40. A	60. D